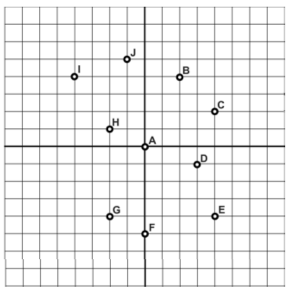
Physics IV
ISI B.Math
Midterm Exam : February 21, 2023

Total Marks: 70
Time : 3 hours
Answer all questions

1. (**Marks : 2 + 2 +2 + 3 + 3 = 12**)



In the space-time diagram above (x, ct) above, each horizontal unit and vertical unit is one light year.

(a) Give one pair of events that are simultaneous in this frame.

(b) Give one pair of events that take place at the same position in this frame.

(c) Give one pair of events that have a light like separation.

(d) List all the events that could be caused by A.

(e) List all the events that could cause A

2. (**Marks : 3 x 5 = 15**)

State whether the following statements are true or false, accompanied by a very brief (one or two lines) justification.

(a) Events A and B occur at the same place in an inertial frame, with A happening before B. It follows that A will happen before B in every inertial frame.

(b) The density of a body will be the same in every inertial frame.

(c) Two clocks at the ends of a train are synchronized with respect to the train. If the train moves past you, the clock in the front shows a higher time.

(d) A particle of mass M decays into two lighter particles of mass m_1 and m_2. Then we must have $M = m_1 + m_2$.

(e) The sum of two null vectors can be a timelike vector.

3. (**Marks : 3 + 3 + 3 + 6 = 15**)

A train of proper length L moves at speed v_1 with respect to the ground. A passenger runs from the back of the train to the front with a speed v_2 with respect to the train. What is the distance covered and the time taken by the passenger

(i) In the train frame ?

(ii) In the ground frame ?

(iii) In a frame of the passenger ?

(iv) Verify that the invariant interval is the same in all frames

4. **Marks: 8 + 4 + 3 + 3 = 18**

a) A photon collides with a stationary electron. If the photon scatters at an angle θ ,use the conservation of 4-momentum to show that the resulting wavelength λ' is given in terms of the original wavelength λ of the photon, by

$$\lambda' = \lambda + \frac{h}{mc}(1 - \cos\theta)$$

where m is the mass of the electron. Note: The energy of a photon is $E = h\nu = \frac{hc}{\lambda}$.

b) Show that the sum of any two orthogonal spacelike vectors is spacelike.

c) Show that a timelike vector and a null vector cannot be orthogonal.

d) Show that the four velocity u must be orthogonal to the corresponding four force F

5. (**Marks : 4 + 3 + 3 = 10**)

(a) An observer in inertial frame S measures a charge density ρ and current density $\mathbf{j}$ in his frame. An observer in frame S' moving with a velocity v with respect to S along the common $x - x'$ axis measures a charge density ρ' and current density $\mathbf{j}'$ in his frame for the same charge and current distribution. How are the quantities $\rho', \mathbf{j}'$ related to the corresponding quantities in the S frame ? Which combination of ρ and $\mathbf{j}$ remains invariant under a Lorentz transformation ? Is it possible for observers to disagree on whether a charge distribution produces only a charge density and no current density ? [Hint: think of how charge density and current density can be grouped into components of a four vector].

The electric and magnetic fields $(\mathbf{E}, \mathbf{B})$ are measured with respect to an observer in an inertial frame S . It can be shown that i) $\mathbf{E} \cdot \mathbf{B}$ and ii) $E^2 - c^2B^2$ are invariant quantities under Lorentz transformations, where E and B represent the magnitudes of the electric and magnetic field respectively.

(b) Show that a pure electric field in one inertial frame cannot be transformed into a pure magnetic field in another inertial frame.

(c) A particular electromagnetic field has its $\mathbf{E}$ field at an angle θ_0 to its $\mathbf{B}$ field, and θ_0 is invariant to all observers. What is the value of θ_0 ?